MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

SUPPLEMENTARY EXAMINATION

TRIMESTER 1, 2015/2016

DIM5068 – MATHEMATICAL TECHNIQUES 2

(For Diploma Students Only)

18 NOV 2015
2.30 PM – 4.30 PM
(2 HOURS)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 2 pages excluding cover page and appendix.
2. Attempt ALL **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.
4. Key formulae are given in the Appendix.

Please answer ALL questions and show the necessary working steps. Each question is 20 marks.

Question 1

a. Simplify the given expression, $(4+2i)^2 - 7(3i+1)$. (2 marks)

b. Given $z = 18(\cos 60^\circ + i\sin 60^\circ)$. Write an expression for z in rectangular form. (5 marks)

c. Find the complex zeros of polynomial function $f(x) = 2x^3 - 3x^2 + 18x - 27$ and write the answer in factored form. (5 marks)

d. Evaluate the following limits.

i. $\lim\limits_{h\to 4} \dfrac{h-4}{\sqrt{h}-2}$ (4 marks)

ii. $\lim\limits_{x\to\infty} \dfrac{4+x^2}{5x^2-3x+1}$ (4 marks)

[TOTAL 20 MARKS]

Question 2

a. Find the derivatives of the function, $y = \dfrac{x\sqrt{x^2+1}}{(x+1)^{\frac{2}{3}}}$. (10 marks)

b. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume? (10 marks)

[TOTAL 20 MARKS]

Question 3

a. Find the integral

i. $\int_0^5 (2x^3 + 6x^2 - 3x + 1)\,dx$ (2 marks)

ii. $\int 7t^2 e^t\, dt$ (8 marks)

b. Find the volume of the solid generated when the region enclosed by $x = y + 2$ and $y = x^2 - 2$ is revolved about the x-axis. (10 marks)

[TOTAL 20 MARKS]

Continued…….

Question 4

a. Solve the differential equation $5\dfrac{dp}{dq}=\dfrac{p\left(q^3-9q\right)}{q}$ by using **separable method**. (4 marks)

b. For the differential equation $x^2\dfrac{dy}{dx}+4xy=\dfrac{\sin x}{x^2}-3x^3$, prove that the solution is $y=-\dfrac{\cos x}{x^4}-\dfrac{x^2}{2}+C$. [Hint: use **method of integrating factors**, μ] (10 marks)

c. Find the **general solution** of the differential equation $2y''-4y'+5y=0$. (6 marks)

[TOTAL 20 MARKS]

Question 5

a. In Cartesian coordinates, vector $\vec{A}$ is directed from the origin to point $Q_1 = (-2, 0, -6)$, and vector $\vec{B}$ is directed from Q_1 to point Q_2 (0, 4, -2). Find

   i. vector $\vec{A}$ (1 mark)

   ii. vector $\vec{B}$ (2 marks)

   iii. unit vector $\hat{a}$ (3 marks)

   iv. the angle between $\vec{A}$ and $\vec{B}$ (5 marks)

b. Given points $A=(2,1,0), B=(3,5,7)\ and\ C=(4,3,10)$.

   i. Find a vector orthogonal to the plane through points *A*, *B* and *C*. (7 marks)

   ii. Find the area of the triangle ABC. (2 marks)

[TOTAL 20 MARKS]

End of page.

APPENDIX

Derivatives: $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}(x^n)=nx^{n-1}$
4. $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}(e^x)=e^x$
2. $\frac{d}{dx}(a^x)=a^x\ln a$
3. $\frac{d}{dx}(\ln x)=\frac{1}{x}$
4. $\frac{d}{dx}(\log_a x)=\frac{1}{x\ln a}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x)=\cos x$
2. $\frac{d}{dx}(\cos x)=-\sin x$
3. $\frac{d}{dx}(\tan x)=\sec^2 x$
4. $\frac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\frac{d}{dx}(\sec x)=\sec x\tan x$
6. $\frac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\frac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\frac{du}{u}=\ln|u|+C$
4. $\int e^u\,du=e^u+C$
5. $\int\sin u\,du=-\cos u+C$
6. $\int\cos u\,du=\sin u+C$
7. $\int\sec^2 u\,du=\tan u+C$
8. $\int\csc^2 u\,du=-\cot u+C$
9. $\int\sec u\tan u\,du=\sec u+C$
10. $\int\csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b[f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\, dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos \theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$

Area for parallelogram *PQRS*

$= \left| \vec{PQ} \times \vec{PR} \right|$

Area for triangle PQR

$= \dfrac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\cos \theta = \dfrac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$